Integrating Logical and Sub-symbolic Contexts of Reasoning[1]

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Abstract

We propose an extension of the heterogeneous multi-context reasoning framework by G. Brewka and T. Eiter, which, in addition to logical contexts of reasoning, also incorporates sub-symbolic contexts of reasoning. The main findings of the paper are a simple extension of the concept of bridge rules to the sub-symbolic case and the concept of bridge rule models that allows for a straightforward enumeration of all equilibria of multi-context systems.

1 INTRODUCTION

An important problem in knowledge representation and knowledge engineering is the impossibility of writing globally true statements about realistic problem domains. A circumstance that is also documented by the common use of contexts and micro-theories. Multi-context systems (MCS) are a formalization of simultaneous reasoning in multiple contexts . Different contexts are inter-linked by bridge rules which allow for a partial mapping between formulas/concepts/information in different contexts. Recently there have been a number of investigations of MCS reasoning (e.g. [4] or [2]), with [1] being one of the latest contributions. There, the authors describe reasoning in multiple contexts that may use different logics locally. Logical reasoning on the one hand is a special case of symbolic reasoning where, according to [3], entities of the application domain are represented by symbols. In sub-symbolic reasoning on the other hand domain entities are represented by (micro-)features.

There is no strict boundary between symbolic and sub-symbolic: what in one example are micro-features can be declared as entities and be symbolically reasoned about in another example (and vice versa). In this paper, we integrate contexts of logical reasoning and contexts of sub-symbolic reasoning into a single MCS. Possible applications of such reasoners are numerous. For instance, shortcomings of statistical methods could be remedied with declarative knowledge and vice versa.

2 Integrating Logical and Sub-symbolic Contexts of Reasoning

Departing from the theory stated in [1], we developed the notion of generalized symbolic/sub-symbolic MCS by introducing the crucial concept of a "reasoner" as a replacement for the narrower concept of a "logic" as used by Brewka and Eiter.

Definition 1 *A* reasoner *is a 5-tuple* $R = (\mathbf{Inp}_R, \mathbf{Res}_R, \mathbf{ACC}_R, \mathbf{Cond}_R, \mathbf{Upd}_R)$ *where* $\mathbf{Inp}_R$ *is the set of possible* inputs *to the reasoner,* $\mathbf{Res}_R$ *is the set of possible* results *of the reasoner,* $\mathbf{ACC}_R : \mathbf{Inp}_R \mapsto 2^{\mathbf{Res}_R}$ *defines the actual reasoning (assigning each input a set of results in a decidable manner),* $\mathbf{Cond}_R$ *is a set of decidable conditions on inputs and results,* $cond_R : \mathbf{Inp}_R \times \mathbf{Res}_R \mapsto \{0,1\}$*, and* $\mathbf{Upd}_R$ *is a set of update functions for inputs,* $upd_R : \mathbf{Inp}_R \mapsto \mathbf{Inp}_R$.

[1]The corresponding paper was first published in: Joaquim Filipe, Ana L. N. Fred, Bernadette Sharp (Eds.): *ICAART 2010 - Proceedings of the International Conference on Agents and Artificial Intelligence, Volume 1 - Artificial Intelligence, Valencia, Spain, January 22-24, 2010.* INSTICC Press, 2010, pp. 494-497.

This generalized notion additionally covers sub-symbolic reasoners as e. g. Neural Nets. Using the concepts introduced in Definition 1, we were able to also carry over the other relevant notions from [1] (as e. g. bridge rules and belief states, up to the concept of equilibrium of an MCS) to the symbolic/sub-symbolic case, adding auxiliary theoretical relining where needed. Conclusively we showed that the newly introduced notion of generalized MCS in fact is a generalization of the heterogeneous nonmonotonic MCS.

3 Computing Equilibria for Finite MCS

After elaborating shortly on a possible expansion or a substitute for the concept of minimal equilibria of logical only MCS when dealing with mixed systems containing both symbolic and sub-symbolic contexts, the second main aspect of the paper is addressed: Using the newly developed theory of generalized MCS, we designed an algorithm for finding the equilibria of finite[2] MCS of the generalized type. As the algorithm is based on complete enumeration and testing, we moreover had to introduce some theoretical tools, amongst which the concept of a "bridge rule model" – offering a possibility to expatiate on the actual reasoning by signalizing explicitly which bridge rules within an MCS are active with respect to a given belief state – is the most important one.

Definition 2 *Let* $\mathbf{Br}$ *be a set of n bridge rules of an MCS. A bridge rule model is an assignment* $\mathbf{Br} \mapsto \{0,1\}^n$ *that represents for each bridge rule in* $\mathbf{Br}$ *whether it is active or not.*

Using bridge rule models, we may give a complete and correct algorithm for finding the equilibria of a finite MCS: For a given bridge rule model and an MCS we first apply all the bridge rules activated in the bridge rule model yielding $inp'_1...inp'_n$. Then we compute the set of results for each context i given inp'_i by applying $ACC(inp'_i)$, yielding a set of results res_i^j for each i, being of finite cardinality as MCS was said to be finite. Thus, testing whether (inp_i, res_i^j) is an equilibrium for all j, we obtain the set of equilibria for the given bridge rule model. Iterating the procedure over the (finite) set of all bridge rule models and joining the resulting sets of equilibria finally yields the set of all equilibria.

4 CONCLUSION

The paper presents a generalization of heterogeneous multi-context systems that allows for the use of sub-symbolic contexts of reasoning alongside logical contexts of reasoning. An exhaustive algorithm for enumerating all equilibria of an MCS is given.

Still, the lack of a conceptual notion of minimality or stability for sub-symbolic beliefs poses a challenge for future research. Moreover, on the pragmatic side, testing examples have shown that a more powerful language to describe updates and conditions on reasoner inputs and results, respectively, has to be developed in order to allow for concise definitions of bridge rules. Also, concerning a concise and sound semantics for mixed symbolic/sub-symbolic MCS, a proposal is still in the very early stages of development.

References

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[2] An MCS is finite if both the cardinality of the set of bridge rules, and the cardinality of the set of acceptable results for the input under consideration, are finite. From the application point of view, finite MCS are the most considerable subclass of the set of all MCS.