Coalition Description Logic with Individuals[1]

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1 Introduction

Coalition logic (**CL**) [2] formalizes the ability of groups of agents to achieve certain outcomes in strategic games. On the other hand, description logics (DLs) are logical formalisms for representing the knowledge of an application domain in a structured way [1]. The importance of DLs lies in the fact that they are decidable fragments of first-order logic and they have well developed practical decision procedures. Moreover, they comprise the formal basis of the Semantic Web ontology languages. In [3], we proposed a product style combination of the description logic $\mathcal{ALC}$ with coalition logic, that allowed for application of modal operators to both formulas and concepts. Still, the combination kept the agent and the concept layers pretty much separated; in particular, one could not use the first-order elements of DL to specify how agents and their groups can influence *themselves*. In this paper, we make the first step to overcome the drawback: we extend the language of concepts with names of individuals (including agents), and we allow for more complex terms to define coalitions. This simple extension allows to express surprisingly sophisticated properties. Furthermore, we extend the satisfiability procedure from [3] to handle the new language, and we establish complexity bounds for the satisfiability problem.

This is an extended abstract of a paper that appeared in *Electronic Notes in Theoretical Computer Science*. For more technical details, please refer to the original paper.

2 The Logic

In order to specify properties of agents and coalitions, we assume the following sets of names: a countable set N_C of *concept names* that includes at least $\mathbb{A}\mathrm{gt}$ (the name for the "grand coalition" of agents), a countable set N_R of *role names*, a finite nonempty set N_I of *individual names*, and a finite nonempty set $N_A \subseteq N_I$ of *agent names*. The set of *concepts* is the smallest set satisfying the following conditions:

- $\top$ is a concept (top concept), and every concept name is a concept;
- If $\mathbf{i}_1, \ldots, \mathbf{i}_n$, $n \geq 0$, are individual names then $\{\!|\mathbf{i}_1, \ldots, \mathbf{i}_n|\!\}$ is a concept (enumeration of individuals);
- If C is a concept and R is a role name then $\forall R.C$ and $\exists R.C$ are concepts;
- If C and D are concepts then $C \sqcap D$, $C \sqcup D$, and $C \backslash\!\backslash D$ are also concepts;
- If C is a concept and $\mathbb{A}$ is a coalitional term (defined further) then $[\mathbb{A}]C$ and $\langle\mathbb{A}\rangle C$ are concepts.

A *coalitional term* is a concept that includes only names from $N_A \cup \{\mathbb{A}\mathrm{gt}\}$.

Now we can define the set formulas of $\mathbf{CL}_{\mathcal{ALCO}}$ as follows: if C, D are concepts then $C \sqsubseteq D$ and $C = D$ are (atomic) formulas; if φ, ψ are formulas then $\neg\varphi, \varphi \wedge \psi, \varphi \vee \psi$ are formulas; if φ is a formula and $\mathbb{A}$ is a coalitional term then $[\mathbb{A}]\varphi$, $\langle\mathbb{A}\rangle\varphi$ are also formulas.

A *model for* $\mathbf{CL}_{\mathcal{ALCO}}$ is a quadruple of the form $\mathfrak{M} = \langle \mathrm{Agt}, W, E, \mathcal{I}\rangle$, where Agt is a finite nonempty set of *agents*, W is a nonempty set of *possible worlds* (*states*), and $E, \mathcal{I}$ associate a *playable effectivity function* E_w and an $\mathcal{ALCO}$*-interpretation* $\mathcal{I}(w)$ with every world $w \in W$. The interpretation $\mathcal{I}(w)$ defines

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the denotation of individual names and primitive concepts in state w. We make a number of structural assumptions which ensure that agents are a part of domain in every state, and can be referred to like all other concepts and individuals. Moreover, the interpretation of coalitional terms does not change from state to state, and the cardinality of the set of agents Agt is the same as the number of agent names given in N_A.

$\mathcal{I}(w)$ is extended to complex concepts in a standard way, with $\mathbf{CL}_{\mathcal{ALCO}}$-specific cases defined as follows:

$$\begin{aligned}
\{\!\!\{\mathbf{i}_1, \ldots, \mathbf{i}_n\}\!\!\}^{\mathcal{I}(w)} &= \{\mathbf{i}_1^{\mathcal{I}(w)}, \ldots, \mathbf{i}_n^{\mathcal{I}(w)}\},\\
([\mathbb{A}]C)^{\mathcal{I}(w)} &= \{\delta \in \Delta^{\mathcal{I}(w)} \mid \|C\|_\delta^{\mathfrak{M}} \in E_w(\mathbb{A}^{\mathcal{I}(w)})\},\\
(\langle\mathbb{A}\rangle C)^{\mathcal{I}(w)} &= \{\delta \in \Delta^{\mathcal{I}(w)} \mid W \setminus \|C\|_\delta^{\mathfrak{M}} \notin E_w(\mathbb{A}^{\mathcal{I}(w)})\},
\end{aligned}$$

where $\|C\|_\delta^{\mathfrak{M}} = \{w \in W \mid \delta \in C^{\mathcal{I}(w)}\}$ is the set of states that δ belongs to the interpretation of concept C.

Now we can use the following semantic clauses for the modal part of $\mathbf{CL}_{\mathcal{ALCO}}$:

$$\begin{aligned}
\mathfrak{M}, w \models [\mathbb{A}]\varphi &\quad \text{iff} \quad \|\varphi\|^{\mathfrak{M}} \in E_w(\mathbb{A}^{\mathcal{I}(w)}),\\
\mathfrak{M}, w \models \langle\mathbb{A}\rangle\varphi &\quad \text{iff} \quad W \setminus \|\varphi\|^{\mathfrak{M}} \notin E_w(\mathbb{A}^{\mathcal{I}(w)}),
\end{aligned}$$

where $\|\varphi\|^{\mathfrak{M}} = \{w \in W \mid \mathfrak{M}, w \models \varphi\}$ is the set of states that satisfy φ in $\mathfrak{M}$.

Example 1 Let $Perm$ stand for the set of permissions to be in a building, and In represent the set of agents that are currently inside. Formula $\bigwedge_{\mathbf{a} \in N_A} \big([\mathbf{admin}](\{\!\!\{\mathbf{a}\}\!\!\} \sqsubseteq Perm) \wedge [\mathbf{admin}]\neg(\{\!\!\{\mathbf{a}\}\!\!\} \sqsubseteq Perm)\big)$ specifies that the administrator can grant and deny the permission to any agent. Moreover, $\bigwedge_{\mathbf{a} \in N_A} \neg(\{\!\!\{\mathbf{a}\}\!\!\} \sqsubseteq In) \rightarrow \big(\{\!\!\{\mathbf{a}\}\!\!\} \sqsubseteq Perm \leftrightarrow [\mathbf{a}](\{\!\!\{\mathbf{a}\}\!\!\} \sqsubseteq In)\big)$ says that an agent is able to enter the building if, and only if, he has a permission to do so. ■

Example 2 Consider a system with a dynamic hierarchy of processes captured by the $parent$ role name, and a dynamic configuration of active processes represented by the concept $Active$. Formula $\bigwedge_{\mathbf{a} \in N_A} (Active \sqcap [\mathbf{a}]\neg Active) = \exists parent.\{\!\!\{\mathbf{a}\}\!\!\}$ says that agents can deactivate exactly those processes they are parents of. Adding a requirement that an agent cannot activate a process without becoming its parent: $\bigwedge_{\mathbf{a} \in N_A} \neg Active \sqcap [\mathbf{a}](Active \sqcap \neg\exists parent.\{\!\!\{\mathbf{a}\}\!\!\}) = \bot$ makes for a viable specification of a hierarchic multi-process system. ■

3 Tableau Algorithm for $\mathrm{CL}_{\mathcal{ALCO}}$

Theorem 1 *A $\mathbf{CL}_{\mathcal{ALCO}}$ formula φ is satisfiable iff there exists a compact Hinttika quasistructure for φ.*

From Theorem 1, an algorithm which constructs a (finite) representation of a compact Hintikka quasistructure for a $\mathbf{CL}_{\mathcal{ALCO}}$-formula can be used as a decision procedure for the satisfiability of $\mathbf{CL}_{\mathcal{ALCO}}$-formulas. We describe such an algorithm, and we prove its termination, soundness, and completeness (see the original paper for details).

Theorem 2 *The tableau algorithm runs in* NEXPTIME.

As a consequence, we have that the satisfiability problem for $\mathbf{CL}_{\mathcal{ALCO}}$ is in NEXPTIME. We also conjecture that the problem is NEXPTIME-complete.

References

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